Kitaev Model and Majoranas

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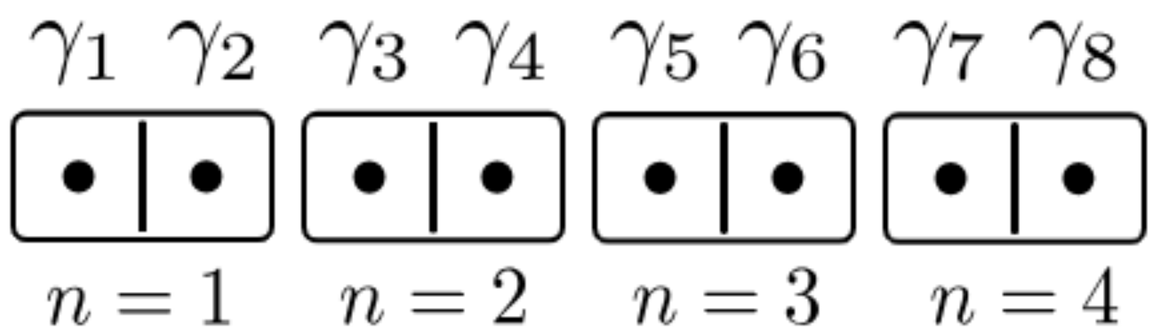
Overview

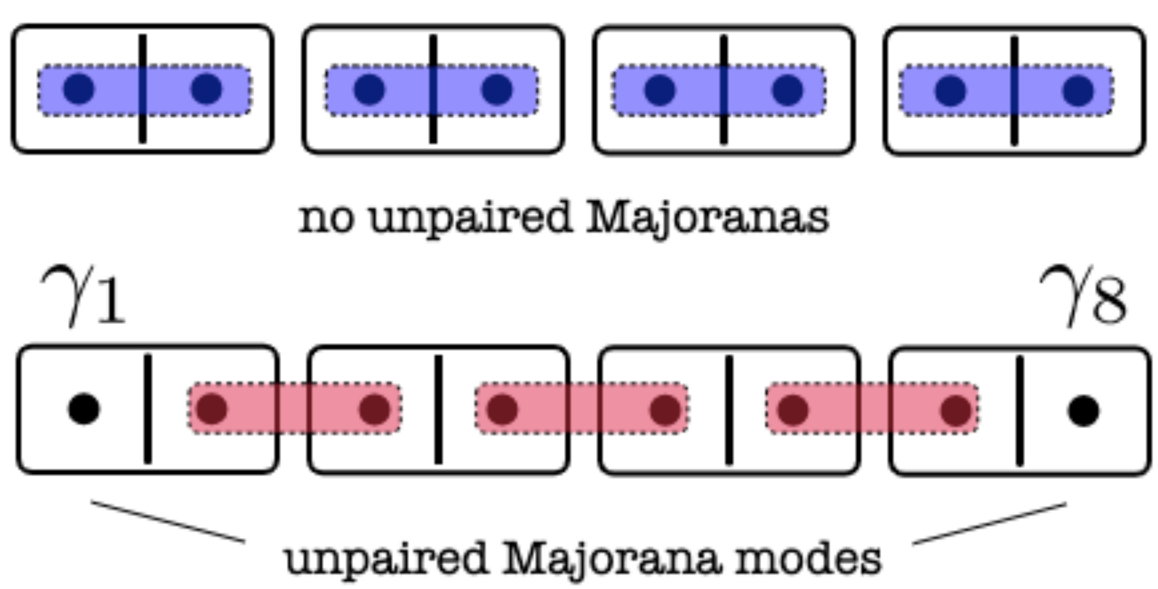
- Majoranas as a transformation
- 1-D Kitaev Hamiltonian
- Kitaev Energies and Topology
- Applications
- 2-D Kitaev on a Honeycomb Lattice

Majoranas

Imagine if we have a regular set of fermionic operators C and $C^\dagger$ which we transform into a different set according to the rules $C^\dagger = \frac{1}{2}(\gamma_1 + i\gamma_2)$ and $C = \frac{1}{2}(\gamma_1 - i\gamma_2)$

$$\boxed{\{\gamma_1, \gamma_2\} = 0,\ \gamma_1^2 = 0, \gamma_2^2 = 1}$$

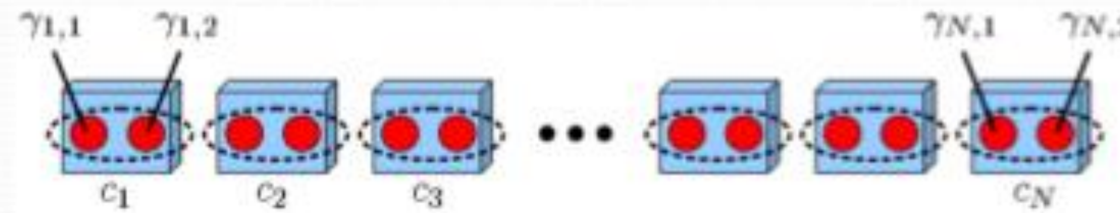




$$H = it \sum_{n=1}^{N-1} \gamma_{2n}\gamma_{2n+1}.$$

$$H = (i/2)\,\mu \sum_{n=1}^{N} \gamma_{2n-1}\gamma_{2n}.$$

Kitaev 1-D Chain



- **Hamiltonian in terms of Majorana Operators**
  - Simple case $\mu = 0,\ t = \Delta$

$$\mathcal{H}_{\text{chain}} = -t\sum_{i=1}^{N-1}\left(c_i^\dagger c_{i+1} + c_i c_{i+1} + c_{i+1}^\dagger c_i + c_{i+1}^\dagger c_i^\dagger\right) \qquad \boxed{\gamma_{i,2} = c_i^\dagger + c_i}$$

$$= -t\sum_{i=1}^{N-1}\left(\left(c_i^\dagger + c_i\right)c_{i+1} + c_{i+1}^\dagger\left(c_i + c_i^\dagger\right)\right) = -t\sum_{i=1}^{N-1}\left(\gamma_{i,2}c_{i+1} + c_{i+1}^\dagger\gamma_{i,2}\right)$$

$$= -\frac{t}{2}\sum_{i=1}^{N-1}(\gamma_{i,2}\gamma_{i+1,2} + i\gamma_{i,2}\gamma_{i+1,1} + \gamma_{i+1,2}\gamma_{i,2} - i\gamma_{i+1,1}\gamma_{i,2})$$

  - Recall anticommutation $\{\gamma_{i,k}, \gamma_{j,\ell}\} = 2\delta_{ij}\delta_{k\ell}$

$$\mathcal{H}_{\text{chain}} = -\frac{t}{2}\sum_{i=1}^{N-1}(\gamma_{i,2}\gamma_{i+1,2} + i\gamma_{i,2}\gamma_{i+1,1} - \gamma_{i,2}\gamma_{i+1,2} + i\gamma_{i,2}\gamma_{i+1,1}) = \boxed{-it\sum_{i=1}^{N-1}\gamma_{i,2}\gamma_{i+1,1}}$$

Kitaev Hamiltonian

The most general Hamiltonian is:

$$H = -\mu \sum_n c_n^\dagger c_n - t \sum_n (c_{n+1}^\dagger c_n + \text{h.c.}) + \Delta \sum_n (c_n c_{n+1} + \text{h.c.})\,.$$

Onsite Potential **Intersite Hopping** **Superconducting Pairing, think in terms of Cooper Pairs**

If we now have,

$$C \equiv (c_1, \ldots, c_n, c_1^\dagger, \ldots, c_n^\dagger)$$

$$\mathcal{H} = C^\dagger H_{BdG} C$$

Then,

$$H_{BdG} = -\sum_n \mu \tau_z \left|n\right\rangle \left\langle n\right| - \sum_n \left[(t\tau_z + i\Delta\tau_y)\ \left|n\right\rangle \left\langle n+1\right| + \text{h.c.}\right].$$

This is the Boguliubov de-Gennes Trick!

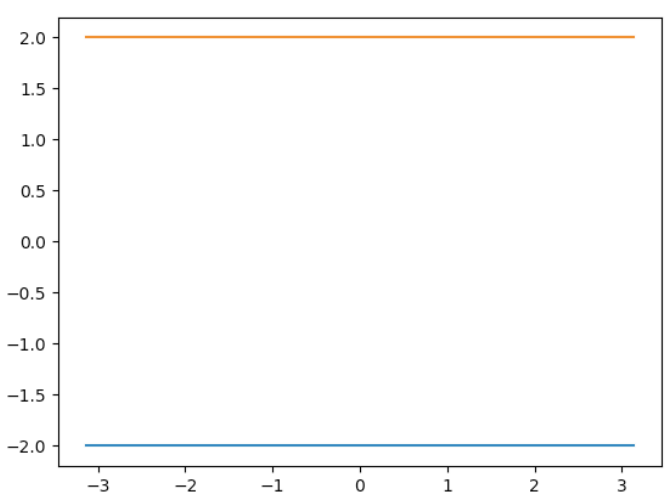
Fourier Space

$$|k\rangle = (N)^{-1/2} \sum_{n=1}^{N} e^{-ikn} |n\rangle \,.$$

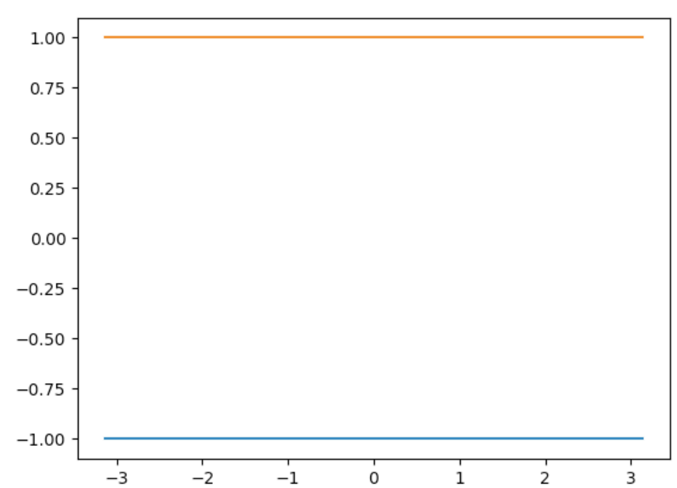
$$H(k) \equiv \langle k| H_{\mathrm{BdG}} |k\rangle = (-2t\cos k - \mu)\,\tau_z + 2\Delta \sin k\; \tau_y.$$

The eigenspectra is thus:

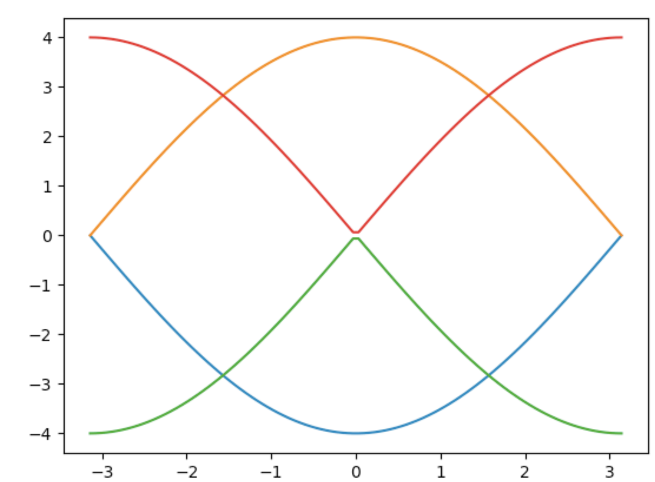
$$E(k) = \pm\sqrt{(2t\cos k + \mu)^2 + 4\Delta^2 \sin^2 k}.$$



$\mu = 0, t = \delta = 1$



$\mu = 1, t = \delta = 0$



$\mu = \pm 2, t = \delta = 1$

Crossing! Topology revealed

Applications

- Robust Information Storage
- Quantum Hall system with filling fraction 5/2
- Core of Half vortices (p-wave superconductors Sr2RuO4)

Read and Green, Phys. Rev. B 61 (2000) 10267;

Ivanov, Phys. Rev. Lett. 86 (2001) 268;

Stern, von Oppen and Mariani, Phys. Rev. B 70 (2004) 205338;

Tewari, Zhang, Das Sarma, Nayak and Lee, Phys. Rev. Lett. 100 (2008) 027001

Quantum Statistics

- Abelian: Single component wavefunction

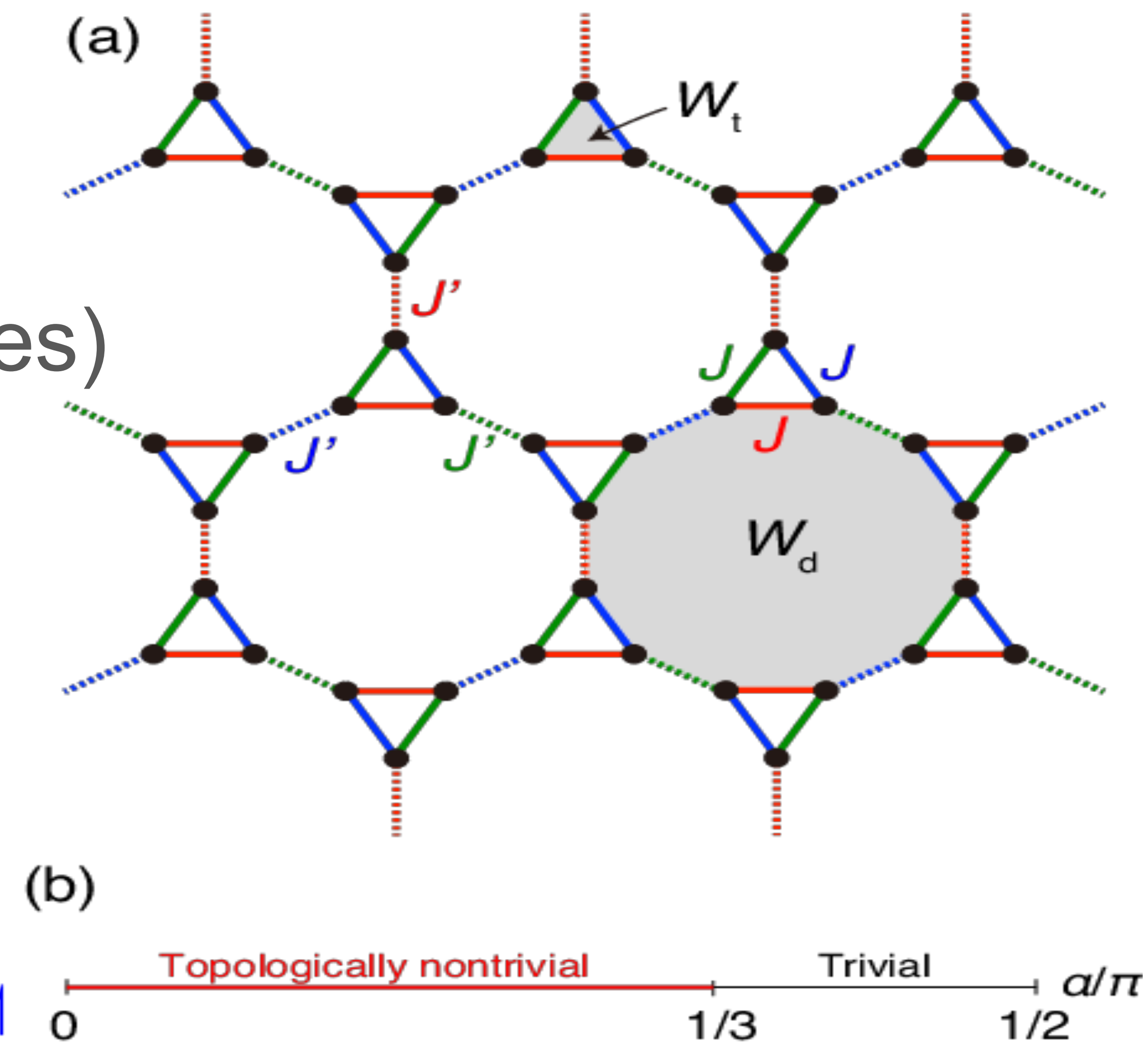
  Phase addition is commutative

- Non-Abelian: N-component wavefunction

  Unitary matrix multiplication is not commutative

Kitaev Model (2D Honeycomb Lattice)

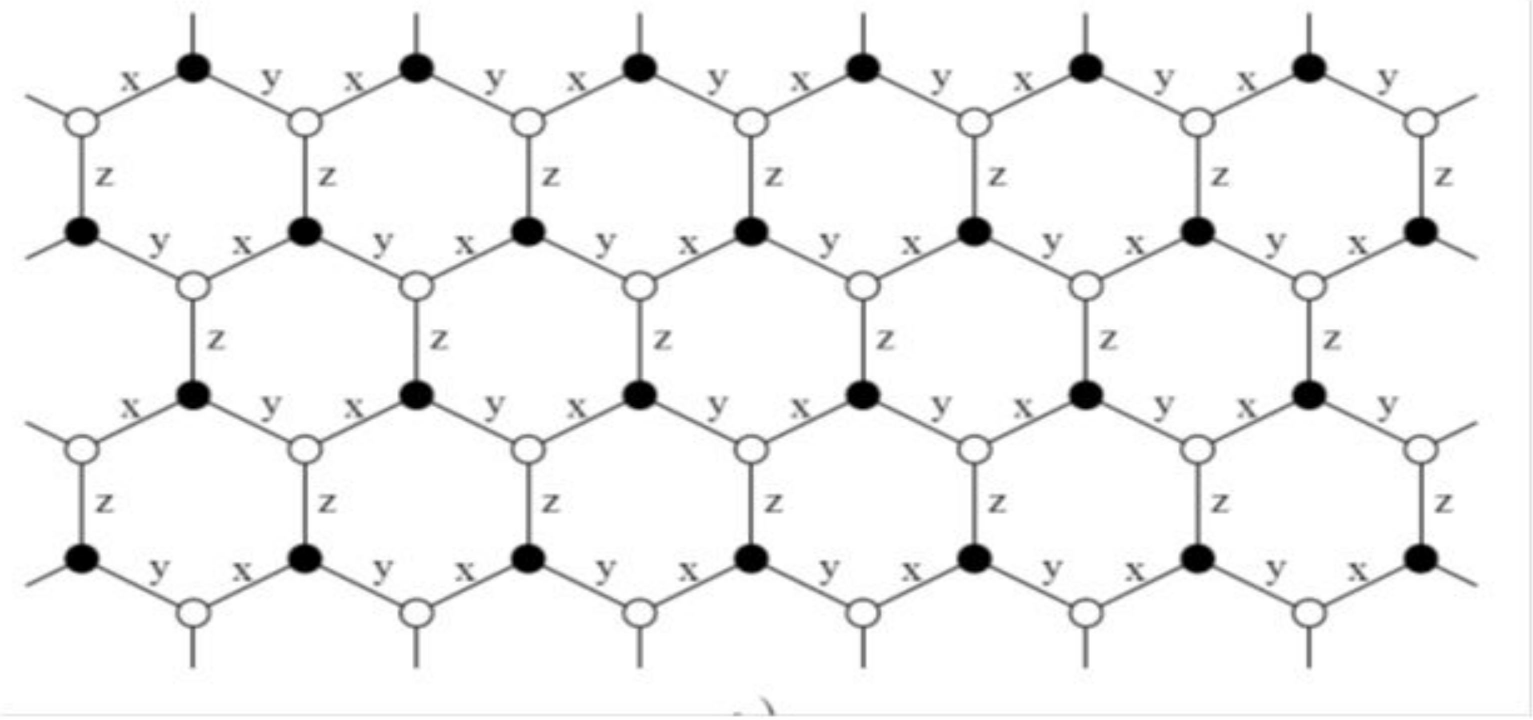
- Representations
- Jordan Wigner Transformation
- ***Conserved Quantities***
- Quantum Statistics (Fusion rules)
- Spin Correlation functions



Feng, Zhang and Xiang, Phys. Rev. Lett. 98 (2007) 087204; Lee, Zhang and Xiang, Phys. Rev. Lett. 99 (2007) 196805; Chen and Nussinov, J. Phys. A 41 (2008) 075001

Kitaev Model

$$H = J_1 \sum_{x-link} \sigma_n^x \sigma_m^x + J_2 \sum_{y-link} \sigma_n^y \sigma_m^y + J_3 \sum_{z-link} \sigma_n^z \sigma_m^z$$



Ground state can be rigorously solved

A. Kitaev, Ann Phys **321**, 2 (2006)

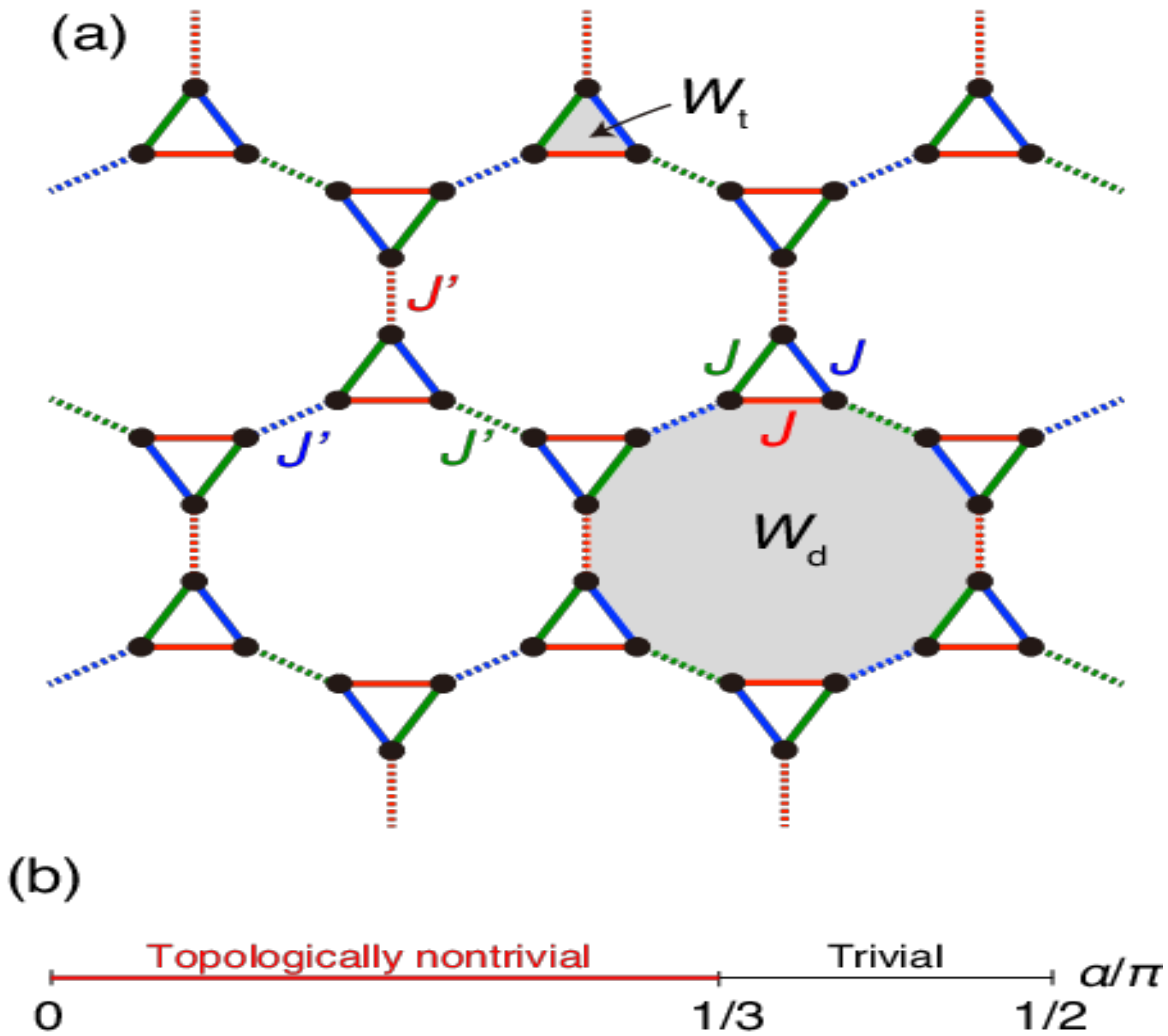
Jordan Wigner Transformation

$$a_{\vec{n}} = \left[\prod_{\vec{m}=-\infty}^{\vec{n}-1} \sigma^z_{\vec{m}} \right] \sigma^y_{\vec{n}} \; (\sigma^x_{\vec{n}})$$

$$b_{\vec{n}} = \left[\prod_{\vec{m}=-\infty}^{\vec{n}-1} \sigma^z_{\vec{m}} \right] \sigma^x_{\vec{n}} \; (\sigma^y_{\vec{n}})$$

Conserved Quantities

$$W = \sigma_1^y \, \sigma_2^z \, \sigma_3^x \, \sigma_4^y \, \sigma_5^z \, \sigma_6^x$$



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Non Abelian Statistics and braiding